

Opener

$$\int \sec^2 x dx =$$

- (A) $\tan x + C$ (B) $\csc^2 x + C$ (C) $\cos^2 x + C$
 (D) $\frac{\sec^3 x}{3} + C$ (E) $2\sec^2 x \tan x + C$

$$\int \frac{5}{1+x^2} dx =$$

- (A) $\frac{-10x}{(1+x^2)^2} + C$ (B) $\frac{5}{2x} \ln(1+x^2) + C$ (C) $5x - \frac{5}{x} + C$
 (D) $5 \arctan x + C$ (E) $5 \ln(1+x^2) + C$

Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}} dx$?

- (A) $\arcsin \frac{x}{5} + C$ (B) ~~$\arcsin x + C$~~ (C) $\frac{1}{5} \arcsin \frac{x}{5} + C$
~~(D) $\sqrt{25-x^2} + C$~~ (E) $2\sqrt{25-x^2} + C$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{25-x^2}} = \frac{5}{\sqrt{25-x^2}} \cdot \frac{1}{5} = \frac{1}{\sqrt{1-(\frac{x}{5})^2}}$$

$$\sqrt{1-\frac{x^2}{25}} = \sqrt{\frac{25-x^2}{25}} = \frac{\sqrt{25-x^2}}{5}$$

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6-1 day 1 Differential Equations

Learning Objectives:

Diffy Q

I can Solve a Differential Equation

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A differential equation is an equation involving a derivative like:

$$\frac{dy}{dx} = 3xy^2$$

Ex1. Solve the differential equation

1.) $\frac{dy}{dx} = x^3 - 2x^2$ given the initial condition $(2, 1)$

$$\int dy = \int (x^3 - 2x^2) dx$$

$$y = \frac{1}{4}x^4 - \frac{2}{3}x^3 + C$$

$$1 = \frac{1}{4} \cdot 2^4 - \frac{2}{3} \cdot 2^3 + C$$

$$1 = 4 - \frac{16}{3} + C$$

$$-3 + \frac{16}{3} = C$$

$$-\frac{9}{3} + \frac{16}{3} = C$$

$$-\frac{9}{3} + \frac{16}{3} = C$$

$$y = \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{7}{3}$$

Steps to Solving a DiffyQ

- 1.) Separate the variables
- 2.) Integrate both sides
- 3.) Solve for y (if possible)
- 4.) Use the initial condition to find C

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2.) $\frac{dy}{dx} = 4x^3 - \frac{2}{x}$ given the initial condition (2,5)

$$\begin{aligned} \int dy &= \int (4x^3 - \frac{2}{x}) dx \\ y &= x^4 - 2\ln x + C \\ 5 &= 16 - 2\ln 2 + C \\ -11 + 2\ln 2 &= C \\ y &= x^4 - 2\ln x + -11 + 2\ln 2 \end{aligned}$$

3.) $\frac{dy}{dx} = 3xy^2$ given the initial condition (2,1)

$$\begin{aligned} \int \frac{y^{-2} dy}{y^2} &= \int 3x dx \\ -1/y^1 &= \left(\frac{3}{2}x^2 - 7\right) \\ -1/y &= \frac{3}{2}x^2 + C \\ -1 &= \frac{3}{2}x^2 + C \\ -7 &= C \\ y &= \frac{-1}{\frac{3}{2}x^2 - 7} \end{aligned}$$

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Ex2. Find the general solution to the differential equation

$$\begin{aligned} \frac{dy}{dx} &= -\frac{x}{y} & y &= \pm \sqrt{-x^2 + C} \\ \int y dy &= -\int x dx \\ \frac{1}{2}y^2 &= -\frac{1}{2}x^2 + C \\ y^2 &= -x^2 + C \end{aligned}$$

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Homework

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